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08EC046

M.Tech. Degree Examination, December 2010
Linear Algebra

Time: 3 hrs. Max. Marks:100

Note: ***1. Answer any FIVE full questions.***
2. Missing data, if any, may be suitably assumed.

1 a. Solve the system of equations
$2x - y + 3z = 2$; $x + 4y = -1$; $2x + 6y - z = 5$. **(06 Marks)**

b. Find the inverse of the matrix
$$A = \begin{bmatrix} 3 & -6 & 3 \\ 6 & -7 & 2 \\ -1 & 7 & 0 \end{bmatrix}.$$ **(07 Marks)**

c. Solve the linear system : $4x_1 + x_2 + x_3 = 4$, $x_1 + 4x_2 - 2x_3 = 4$, $3x_1 + 2x_2 - 4x_3 = 6$ by using LU – decomposition method. Take $u_{ii} = 1$. **(07 Marks)**

2 a. Prove that a non empty sub set W of a vector space V over a field F is a subspace of V if and only if.
i) $\forall\, \alpha, \beta \in W \Rightarrow \alpha + \beta \in W$
ii) $C \in F, \alpha \in W \Rightarrow c . \alpha \in W$. **(06 Marks)**

b. If W, and W_2 are subspaces of the vector space V(F), then prove that $W_1 + W_2$ is a subspace of V(F). **(07 Marks)**

c. Let S be a non empty subset of a vector space V(F), then prove that L(S) is a subspace of V . **(07 Marks)**

3 a. Let V and W be vector spaces over the field F and let T be a linear transformation from V to W. Suppose that V is finite – dimensional. Then, show that rank (T) + nullity (T) = dim (V). **(07 Marks)**

b. Let V be an n-dimensional vector space over – the field F, and let W be an m-dimensional vector space over F. Then show that the space L (V, W) is finite dimensional and has dimension mn. **(07 Marks)**

c. Show that the set $S = \langle (1,1,0), (1,0,1), (0,1,1) \rangle$ is a basis of the vector space $V_3(R)$. **(06 Marks)**

4 a. Let T : V → W be a linear map and d(V) = d(W) = n. Then, show that 'T' is non – singular, iff 'T' → transforms linearly independent vector of V in to linearly independent vectors of W. **(07 Marks)**

b. Show that the linear transformation J : $V_3(R) \to V_3(R)$ is defined by $T(x_1, x_2, x_3) = (x_1 + x_2 + x_3, x_3 - x_2, x_3)$ is non singular and find its inverse. **(07 Marks)**

c. Let T_1 and T_2 be linear operators oñ R^2 defined as follows :
$T_1 (x_1,x_2) = (x_2, x_1)$; $T_2 (x_1, x_2) = (x_1, 0)$. Show that $TT_2 \neq T_2T_1$. **(06 Marks)**

5 a. Let T : V → W be a linear transformation. Then prove that R(T) is a subspace of W. **(06 Marks)**

b. Find the matrix of the linear transformation T : $V_3(R) \to V_2(R)$ defined by T(x, y, z) = (x + y, y + z) relative to the bases B_1 = [(1, 1, 1), (1, 0, 0), (1, 1, 0)], B_2 = [(1, 0), (0, 1)]. **(07 Marks)**

c. Let T : V → W be a linear transformation defined by T(x, y, z) = (x + y, x – y, 2x + z). Find the range, null space, rank, nullity and hence verify the rank nullity theorem. **(07 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

6 a. If $S = \langle u_1, u_2 \text{----} up \rangle$ is an orthogonal set of non zero vectors in R^n. Then prove that S is linearly independent and hence is a basis for the subspace spanned by S. **(06 Marks)**

b. The set $S = \langle u_1, u_2, u_3 \rangle$ [u_1, u_2, u_3] where $u_1 = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$, $u_2 = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$ $u_3 = \begin{bmatrix} -1/2 \\ -2 \\ 7/2 \end{bmatrix}$ is an orthogonal basis of R^3. Express the vector $y = \begin{bmatrix} 6 \\ 1 \\ -8 \end{bmatrix}$ as a linear combination of the vectors in S. **(07 Marks)**

c. Find a QR factorization of $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. **(07 Marks)**

7 a. Show that a linear operator 'T' on a finite dimensional inner product space V is unitary iff 'T' preserves inner products. **(06 Marks)**

b. If $T : V \rightarrow W$ be a non – singular linear map then prove that $T^{-1} : W \rightarrow V$ is linear and bijectives. **(07 Marks)**

c. Find the least square solution of the system Ax =b

for $A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}$; $b = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$. **(07 Marks)**

8 a. Orthogonally diagonalize the matrix

$A = \begin{bmatrix} 3 & -2 & 4 \\ -2 & 6 & 2 \\ 4 & 2 & 3 \end{bmatrix}$. **(07 Marks)**

b. Find the singular value decomposition of

$A = \begin{bmatrix} 1 & 11 & 14 \\ 8 & 7 & -2 \end{bmatrix}$. **(07 Marks)**

c. Find the maximum and minimum values of $Q(x) = 9x_1^2 + 4x_2^2 + 3x_3^2$ subject to the constraint $x^T x = 1$. **(06 Marks)**

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